**2010 IMSA Junior High Math Competition**

7th Grade Individual Contest

1. If the average of Jon's five test scores is 85 and he got 90, 100, 85, and 75 on the first four tests, what was his score on the final exam? (All the test scores are out of 100, and they're all weighted equally.)

2. Samir needs a matching pair of socks. He has 8 green socks, 16 red socks, 18 purple socks, and 2 pink socks (so he has 4 pairs of green socks, 8 pairs of red, etc.), but the problem is that the lights are off in his room. How many single socks does he need to pull out of his drawer in order to guarantee a matching pair?

3. There are 100 closed doors in a hallway, and they're all marked, in order from left to right, with the numbers from 1 to 100 (so the leftmost door has a "1" and the rightmost door has "100"). Omotayo runs down the hallway and *opens* all of the doors that are marked with numbers divisible by 1 so that all of them are now open. Then, she runs back and *closes* all doors with numbers divisible by 2, and she continues this process (closing open doors and opening closed doors) for every number up to 100. How many doors are open when she's done?

4. Sara bought some pieces of chocolate and some pieces of candy for 51 cents and 9 cents apiece respectively. If she spent $2.16 on a total of 10 items, how many pieces of chocolate did she buy?

5. How many different ways can Mrs. O'Leary choose groups of three out of her class of 12 students?

6. If the sum of two numbers is 14 and their product is 48, what is the sum of their squares?

7. Jayanshu needs to find the number with the greatest number of divisors among a list that her math teacher gave her. She correctly identifies 1690 as the answer, but she incorrectly writes "42" down as the number of positive factors 1690 has on her homework. How many factors (also known as divisors) does 1690 actually have?

8. A train moving at 75mph can get from Chicago, Illinois, to St. Louis, Missouri, in 4 hours. How long will it take a train moving at 60mph to get from Chicago to St. Louis?

9. In a community of trolls, there are two clans; members of one always tell the truth, and members of the other always lie. A quartet of trolls, Alina, Bonny, Corinne, and David, sits around a campfire, sharing statements.

   ALINA: I always tell the truth, but Bonny rarely does.
   BONNY: I always tell the truth, but Alina rarely does.
   CORINNE: If Alina is lying, Bonny is also lying.

DAVID: Corinne never tells the truth.

Knowing that two of the trolls always tell the truth and two of the trolls always lie, is David a liar?

10. Find the last digit of $7^{2009}+8^{2010}+25^{2011}$.

11. What is the greatest prime factor of the sum of all the numbers from 1 to 200?

12. The science wing at IMSA needs repainting. If Nolan can paint one-fifth of the wing in an hour and a half, and his friend Webster can paint one-sixth of the wing in an hour, how many hours will it take for the wing to be painted if the two friends work together? Express your answer, in hours, as an improper fraction.

13. The menu at a restaurant consists of four appetizers, two entrées, and seven side dishes. A special deal is offered to the customers who order one appetizer, one entrée, and three side dishes. How many different meals qualify for this deal?

14. In a regular octagon, find the degree measure of an interior angle.

15. Let an *odd date* be a date that is comprised of only odd digits (e.g. 11/13/1933 is an *odd date*, but 03/17/1997 is not an odd date as "0" is an even number). If the last odd date occurred in November 1999 (11/19/1999), how many full years will pass between that date and the next odd date?

16. What is the smallest whole number $n$ that makes $n^2 - n + 41$ a composite number?

17. A gift shop sells keychains in packs of 7 and 13. What's the largest number of keychains that a person can't obtain by buying these packs?

18. The distance between any two bases on a baseball diamond (which is a square) is 90 feet. If Ivan takes an entire minute to run from home to first and then from first to second, how long will it take him to run directly from home to second (without touching first)? Record your answer in seconds.

19. Farmer Joe wants to put up a fence around a square plot of land. If one foot of fencing costs $10, how much money will it take him to put up a fence around that plot if one side is 10 feet long? Record your answer in dollars.

20. Paul is thinking of a two-digit number, and he bets that Kevin won't be able to guess what it is even if three clues are given. He tells Kevin that when he divides his number by 5, he gets a remainder of 2, and when he divides his number by 2, he gets a remainder of 1, and when he divides his number by 9, he gets a remainder of 6. What is Paul's number?

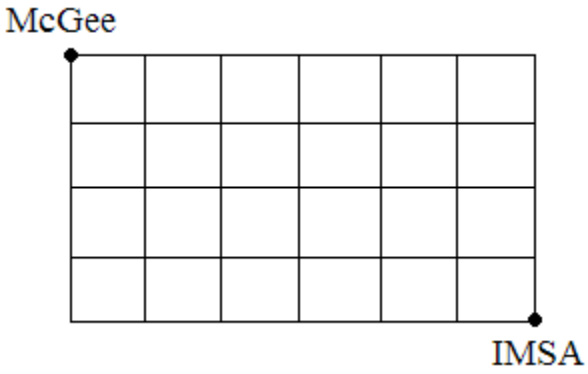
**2010 IMSA Junior High Math Competition**

8th Grade Individual Contest

1. There is a square with perimeter 20. What is the area of this figure?

2. How many ways can the letters that spell ILLINOIS be distinctly rearranged?

3. In the diagram below, Dr. Max McGee starts at a point somewhere to the Northwest of IMSA and walks toward IMSA. If he only walks south or east on the lines one segment at a time, in how many different ways can he get to IMSA?




4. Alex can mow a lawn alone in 4 hours. Alex and his friend Bobby can mow the same lawn together in 1.5 hours. How many hours does it take Bobby to mow the lawn alone?

5. Sally has 4 shirts of different colors (red, blue, green, and yellow) and 3 pairs of pants (red, blue, and black). How many different outfits consisting of one shirt and one bottom can she wear, given that they are not the same color?

6. At how many times from noon to noon are the minute and hour hands of a clock directly opposite one another?

7. Cheng-Po Li is thinking of a two-digit number, and he bets that Kevin won't be able to guess what it is even if three clues are given. He tells Kevin that when he divides his number by 5, he gets a remainder of 2, and when he divides his number by 2, he gets a

remainder of 1, and when he divides his number by 9, he gets a remainder of 6. What is Cheng-Po Li's number?

8. Find the last digit of the product of $3^{2010}$ and $2^{2010}$

9. A line segment is formed by connecting the points at (1,1) and (6,3). Find the distance from (1,1) to the point that divides this segment in half.

10. Irene is writing the solutions for the problems on the Junior High Math Competition at IMSA. As you read this problem, she is writing down the answers to all 85 of the problems. If she can answer 1 question in 2 minutes, how many minutes will it take her to finish this contest?

11. Milly spent $\frac{1}{3}$ of her money on CD's, $\frac{1}{2}$ of her money on posters, and $\frac{1}{10}$ of her money on accessories. If she had a total of \$30 remaining after her purchase, how much money (in dollars) did she spend on accessories?

12. I have 25 coins in my pocket, and the total amount is \$4.00 exactly. The coins are nickels, dimes, and quarters. How many different combinations of coins could I have?

13. The average of Bob's first two test scores is 84, and the average of Cindy's two scores is 98. What is the average score of all four of these scores?

14. Dr. Matsko has an unfair coin that has different probabilities of landing heads and tails. The probability of flipping two straight heads is twice the probability of flipping 2 straight tails. What is the probability of getting heads on a single flip? Express your answer in the form $a-\sqrt{b}$ where *a* and *b* are positive integers.

15. There are 44 red marbles, 56 blue marbles and 3 green marbles in a bag. How many draws will Tim have to make in order to guarantee that he gets at least one marble of every color?

16. There are 25 students in Dr. Kiely's history class. Twenty percent of the students like both math and ice cream, 12 students like math, and 14 like ice cream. Find the probability that a randomly chosen student does not like either math or ice cream.

17. Arrrg! Captain Blackbeard has found a pile of gold coins. When the pile is divided amongst 7 pirates evenly, there are 5 coins left over. When the pile is divided amongst 9 pirates evenly, there are 6 coins left over. What is the smallest number of coins the pile could have contained?

18. In how many ways can you rearrange the letters of "JHMCJHMC" such that the substring "JHMC" does not occur in the shuffled string?

19. A divisor $d$ of a whole number $k$ is a whole number such that $\frac{k}{d}$ is also a whole number. If $k = 23450$, how many divisors does $k$ have?

20. A store sells gum in packs of seven sticks and packs of five sticks. Jane wants to buy a certain number of sticks that is possible by buying a combination of seven and five stick packs. What is the largest possible number of sticks that Jane cannot buy?

**2010 IMSA Junior High Math Competition**

$7^{th}$ Grade Team Contest

1. How many different ways are there to choose one president, two vice presidents, and four secretaries out of 10 distinct applicants?

2. Today is Sara's birthday. Lucy, Sara's roommate, needs to decorate the room before Sara comes back at 5PM from band practice; however, Lucy sleeps in and only starts decorating at 2:30PM. Because she can only decorate one-tenth of the room in half an hour, she won't be able to finish in time. In order to make Sara's birthday surprise perfect, Lucy hires Andrew to help her decorate, but Andrew can only decorate three-twentieths of the room in half an hour. If they work together, how many spare minutes will they have between the time they finish and 5PM?

3. If Katie has five different blouses, ten different skirts, and four different pairs of shoes, how many different outfits (consisting of one blouse, one skirt, and one pair of shoes) can she create?

4. Arjun can run around a 120m track in 120 seconds, and Charles can run around the same track in 100 seconds. If they start running at the same time from the same position on the track, how many seconds will pass before they are running alongside one another again?

5. Nirali has a bunch of dimes and pennies on her desk. If those 24 coins are worth \$0.87, how many dimes does Nirali have?

6. Grace wants to paint the ceiling of a 4ft. by 7ft. room, but she only has \$85. If Nolan and Webster demand \$2 for every square foot that they paint, how much money will Grace have left over? Record your answer in dollars.

7. Fermi wants to get the attention of this girl he likes, but she only likes guys who are good at math. In order to impress her, he needs to buy her a box of chocolates on the $n$-th hour of 2010, assuming that 12AM-1AM on January 1st is the first hour and where $n$ is the only three-digit prime factor of 999,991. On what day should Fermi buy his gift? Express your answer in as a date, such as January 1, 2010.

8. There are 6 more girls than boys in Dr. Condie's class of 24 students. What is the ratio of girls to boys?

9. In the cryptarithm MEMO + FROM = HOMER, every letter stands for a distinct digit between 0 and 9. What is the value of M?

10. At the carnival, three friends win three identical green balls. However, Liana won two while Irene only won one (Grace didn't win any). If Jayanshu steals all of the balls and gives them back

randomly, what is the probability that each girl keeps the same number of balls? Express your answer as a fraction reduced to lowest terms.

11. If Priya has 40 identical sugar cubes and 37 different cups of coffee, how many ways can she put sugar cubes into her coffee? (Every cup of coffee must have some sugar.)

12. Alex has a glass cube with a side length of 4 inches. If he cuts a 2-inch by 2-inch square hole into the center of each face (where the edges of each cut are parallel to the edges of the cube and each cut goes all the way through the cube), what is the volume of the remaining solid?

13. Ben and James are playing an abridged version of tennis where the "must win by two" rule is taken out. (Recall that in tennis a player must win four points - 15, 30, 40, GAME - to win a game.) If Ben has a 2/3 probability of winning a point (because his racket is better) and James has a 1/3 probability of winning a point, what is the chance that James wins the game?

14. The area of a circle is $800\pi$. What is the circumference of that same circle?

15. A palindrome, such as 83438, is a number that remains the same when its digits are reversed. The numbers x and x+32 are three-digit and four-digit palindromes, respectively. What is the sum of the digits of x?

16. IMSA has 185 seniors, each of whom plays on at least one of the school's three varsity sports teams: swimming, tennis, and scholastic bowl. It so happens that 60 are on the swimming team; 124 are on the tennis team; 40 are on both the swimming and scholastic bowl teams; 32 are on both the swimming and tennis teams; and 29 are on both the tennis and scholastic bowl teams. Compute the number of seniors playing all three sports, given that twice this number are members of the scholastic bowl team.

17. If Adam pays $3 as a 15% tip on his order of pizza, how much was the original bill?

18. On an exotic island called Pr01, there are two clans - the Korenos and the Chinos. The Korenos always tell the truth, and the Chinos always lie. There is also a band of outcasts on the island full of natives that tell the truth half of the time and lie the other half of the time. One day, Jesse stumbles on this island and wanders into a camp composed of two Korenos, two Chinos, and one outcast. They make the following statements:

    LAWRENCE: I don't like Eric because he isn't a Koreno. Also, I don't like Karna

    either because he's an outcast.

    BEN: Both David and Eric are Korenos. Lawrence is a Chino.

DAVID: Oh, man, no one likes Ben because he's a Chino. However, I like being around Lawrence, who happens to be a Koreno.

KARNA: Lawrence can dance well, so he must be a Koreno. I'm also a Koreno, so I love to hang out with him.

ERIC: I don't like Lawrence because he isn't a Koreno. Also, I don't like Ben either because he's an outcast.

Who is the outcast?

19. Dr. Fogel wants to make a batch of his legendary Fogel Fudge, but he doesn't have any marshmallows. A pack of seven marshmallows at the store costs $1.50. If he's planning to make 196 servings of fudge and every four servings requires three marshmallows, how much money will he spend buying marshmallows? Express your answer in dollars.

20. Jayanshu is trying to get the pack rat avatar on Neopets, and in order to get it, she needs to collect 1000 different items. If she collects 80 items on the first day, 79 on the second day, 78 on the third day, and so on and so forth, on what day will she get the avatar? Express your answer in the form "DAY ##".

21. Mrs. Harte is grading her $7^{th}$-grade math class's quizzes but seems to have lost the answer key to the ten True/False questions. She knows that:
    a. there are the same number of true answers as false answers
    b. the answers to questions 5 and 9 are false while question 8 is true
    c. the first and last answers are the same
    d. there are more true answers than false answers in the first five questions
    e. there are at least three false answers in a row

    How many possible answer keys meet these criteria?

22. Out of 200 fish in Bella's aquarium, 99% are guppies. How many guppies must be removed so that the percent of guppies in the aquarium is 98%?

23. The product of two positive numbers is 24, and the sum of their squares is 73. What is the square of their difference?

24. From a regular hexagon, three vertices are selected at random. What is the probability that these three vertices form an equilateral triangle or an isosceles triangle? Express your answer as a common fraction.

25. An isosceles trapezoid has bases of 10 units and 20 units and a height of 12 units. What is the perimeter of the trapezoid? An isosceles trapezoid is a trapezoid such that the non-parallel sides have equal length.

26. Which five digit number (or numbers) can you multiply by 4 to get the same number with its digits in reversed order?

27. Suppose that the operation @ is defined as $a@b = a^3 + b^2$. What is the value of $5@(3@4)$?

28. A number consists of three distinct digits. There are six different three digit numbers that can be formed with these three digits. If these six numbers sum up to 5106, find the three digits.

**2010 IMSA Junior High Math Competition**

$8^{th}$ Grade Team Contest

1. If a blind man has a sock drawer with 70 black socks, 70 white socks, and 70 brown socks, how many socks must he pick to guarantee that he has a matching pair?

2. Find the angle between the hour and minute hand of a clock at time 12:47. (Give the exact angle less than $180^\circ$).

3. Samuel has quarters and nickels in his pockets, for a total of 20 coins all together. The value all this change totals $2.60. Find the number of quarters in his pocket.

4. Alex's empty swimming pool will hold 15,000 gallons of water when full. It will be filled by 5 hoses, each of which supplies 3 gallons of water per minute. How many hours will it take to fill this pool?

5. The sum of three positive integers is 37. If one of these integers is 9, what is the greatest possible value of the product of the other two integers?

6. A square with an area of 64 has the same side length as a regular hexagon ( a polygon with six sides of equal length). What is the area of the hexagon?

7. Two angles of an isosceles triangle measure $30^\circ$ and $x^\circ$. What is the sum of all possible values of $x$?

8. How many whole numbers between 1 and 1000 inclusive do not contain the digit 5 or 8?

9. Samantha is preparing to have 37.5% off sale at her clothing store. However, being a sneaky merchant, she wants to increase her regular prices right before the sale so that the sale price is the same as the original price before any price changes are made. By what percent must she raise the prices before the sale to accomplish her goal?

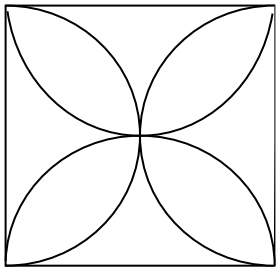
10. Simplify the following complex fraction:

$$\frac{1}{5+\dfrac{1}{5+\dfrac{1}{5}}}+\frac{1}{4+\dfrac{1}{4+\dfrac{1}{4}}}$$

11. What is the smallest positive number can be subtracted from both the numerator and the denominator of 29/34 so the resulting fraction will be equivalent to 4/5?

12. Suppose *a*, *b*, and *c* are positive integers with $3^a 5^b 2^c = 60750$. What is the value of *abc*?

13. The clover below is formed by a square and 4 semicircles. If the side of the square is 6, find the area of the 4 "petals" of the clover.



14. Dr. Condie is taking care of 5 unique babies, and it is play time! If he has 2 distinct playpens, and plans to put all the babies in playpens with at least 1 baby is each pen, in how many different arrangements can he place the babies?

15. If two standard dice are rolled, what is the probability the product of the two numbers rolled is 20 or higher?

16. What is the smallest integer you must multiply 4320 by in order to have a perfect square number?

17. Two numbers have a sum of 528. One of the numbers has a zero as one of its digits. If you remove the zero from this number, then it is equal to the other number. What is the larger of the two numbers that sum to 528?

18. Take any right triangle form a new triangle by connecting the midpoints of the three sides of the right triangle. Repeat this process with the new triangle to form a third triangle. Find the ratio of the area of the smallest triangle to the area of the original right triangle.

19. There is a cylindrical can of 3 spherical tennis balls, each with radius 4cm, touching each other and the sides of the cylinder. How much empty space is there within the container with the balls in it?

20. In the cryptarithm MEMO + FROM = HOMER, every letter stands for a distinct digit between 0 and 9. What is the value of M?